GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP1
0977/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS – FP1

SUMMER 2016 MARK SCHEME

Ques	Solution	Mark	Notes
1	$f(x+h)-f(x)=\frac{(x+h)^2}{(x+h+1)}-\frac{x^2}{x+1}$	**M1A1**	
	$=\frac{(x+h)^2(x+1)-x^2(x+h+1)}{(x+h+1)(x+1)}$	**A1**	
	$=\frac{x^3+x^2+2hx^2+2hx+h^2x+h^2-x^3-hx^2-x^2}{(x+h+1)(x+1)}$	**A1**	
	$=\frac{hx^2+2hx+h^2x+h^2}{(x+h+1)(x+1)}$	**A1**	
	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$		
	$=\lim_{h\to 0}\frac{x^2+2x+hx+h}{(x+h+1)(x+1)}$	**M1**	
	$=\frac{x^2+2x}{(x+1)^2}$	**A1**	
2(a)	The rotation matrix = $\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix}$	**B1**	
	The translation matrix = $\begin{bmatrix}1&0&1\\0&1&2\\0&0&1\end{bmatrix}$	**B1**	
	$\mathbf{T}=\begin{bmatrix}1&0&1\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix}$	**M1**	
	$=\begin{bmatrix}0&-1&1\\1&0&2\\0&0&1\end{bmatrix}$	**A1**	
(b)	The fixed point satisfies $\begin{bmatrix}0&-1&1\\1&0&2\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}x\\y\\1\end{bmatrix}$	**M1**	FT their **T**
	$-y+1=x;\ x+2=y$	**A1**	
	(x,y) = $\left(-\frac{1}{2},\frac{3}{2}\right)$ cao	**m1A1**	

Ques	Solution	Mark	Notes
3	$S_n = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	
	$= \dfrac{n^2(n+1)^2}{4} + \dfrac{n(n+1)(2n+1)}{6}$	A1A1	
	$= \dfrac{n(n+1)(3n(n+1)\ \ +2(2n+1))}{12}$	m1	m1 for attempting to combine and take out two factors
	$= \dfrac{n(n+1)}{12}\left(3n^2+7n+2\right)$	A1	
	$= \dfrac{n(n+1)(n+2)(3n+1)}{12}$	A1	
4(a)	$\lvert z_1 \rvert = 2$; $\arg(z_1) = \dfrac{5\pi}{6}$	B1B1	
	$\lvert z_2 \rvert = \sqrt{2}$; $\arg(z_2) = \dfrac{\pi}{4}$	B1B1	
(b)	EITHER		
	$\lvert w \rvert = \dfrac{\lvert z_1 \rvert^2}{\lvert z_2 \rvert} = \dfrac{4}{\sqrt{2}}$	M1A1	FT from (a)
	$\arg(w) = 2\arg(z_1) - \arg(z_2) = \dfrac{17\pi}{12}$	M1A1	
	$w = \dfrac{4}{\sqrt{2}}\cos\left(\dfrac{17\pi}{12}\right) + \dfrac{4}{\sqrt{2}}\sin\left(\dfrac{17\pi}{12}\right)\mathrm{i}$	M1	
	$= -\ 0.73 - 2.73\mathrm{i}$	A1	
	OR		
	$z_1^{\ 2} = 2 - 2\sqrt{3}\mathrm{i}$	(M1A1)	
	$\dfrac{z_1^{\ 2}}{z_2} = \dfrac{2-2\sqrt{3}\mathrm{i}}{(1+\mathrm{i})} \times \dfrac{1-\mathrm{i}}{1-\mathrm{i}}$	(M1)	
	$= \dfrac{2-2\sqrt{3}-(2\sqrt{3}+2)\mathrm{i}}{2}$	(A1A1)	A1 numerator, A1 denominator
	$= -\ 0.73 - 2.73\mathrm{i}$	(A1)	
	OR		
	$z_1^{\ 2} = 2 - 2\sqrt{3}\mathrm{i}$	M1A1	
	$a + \mathrm{i}b = \dfrac{2-2\sqrt{3}i}{1+\mathrm{i}}$		
	$(a+\mathrm{i}b)(1+\mathrm{i}) = 2 - 2\sqrt{3}\mathrm{i}$	M1	
	$a - b + i(a+b) = 2 - 2\sqrt{3}\mathrm{i}$	A1	
	$a - b = 2; a + b = -2\sqrt{3}$	A1	
	$\dfrac{z_1^{\ 2}}{z_2} = -0.73 - 2.73\mathrm{i}$	A1	

Ques	Solution	Mark	Notes
5(a)(i)	det**M** = $2(\lambda+2)+5(-\lambda)+\lambda(-\lambda^2)$ $= 4-3\lambda-\lambda^3$	M1A1	Or equivalent
(ii)	Substituting λ = 1, det**M** =0 (therefore singular).	B1	
	$4-3\lambda-\lambda^3=(1-\lambda)(\lambda^2+\lambda+4)$	M1A1	Do not accept unsupported answers
	The other two roots (of det**M** = 0) are complex since $b^2-4ac=-15$ so no other real values of λ result in a singular **M**. cao	A1	
(iii)	Using row operations, $\begin{bmatrix}0 & 1 & -1\\ 0 & 1 & -1\\ 1 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$	M1 A1	
	The first two (complete) rows are identical therefore consistent.	A1	
	Let $z = \alpha$. Then $y = \alpha + 1$. and $x = -3\alpha-1$.	M1 A1 A1	
(b)	Now, **M** = $\begin{bmatrix}2 & 5 & -1\\ 0 & -1 & -1\\ -1 & 2 & 1\end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix}1 & 1 & -1\\ -7 & 1 & -9\\ -6 & 2 & -2\end{bmatrix}$	M1 A1	Award M1 if at least 5 elements correct
	Adjugate matrix = $\begin{bmatrix}1 & -7 & -6\\ 1 & 1 & 2\\ -1 & -9 & -2\end{bmatrix}$	A1	
	Det**M** = 8	B1	
	$\mathbf{M}^{-1}=\frac{1}{8}\begin{bmatrix}1 & -7 & -6\\ 1 & 1 & 2\\ -1 & -9 & -2\end{bmatrix}$	A1	FT from adjugate matrix and determinant

Ques	Solution	Mark	Notes
6	Let the roots be $\alpha, \frac{1}{\alpha}, \beta$.	**M1**	
	Then,		
	$\alpha + \frac{1}{\alpha} + \beta = -\frac{b}{a}$ (i)		
	$1 + \alpha\beta + \frac{\beta}{\alpha} = \frac{c}{a}$ (ii)	**A1**	
	$\beta = -\frac{d}{a}$ (iii)		
	From (i), $\alpha + \frac{1}{\alpha} = -\frac{b}{a} + \frac{d}{a}$	**M1A1**	M1 attempting to eliminate one of the parameters
	From (ii), $\alpha + \frac{1}{\alpha} = \left(\frac{c}{a} - 1\right)\left(-\frac{a}{d}\right)$	**A1**	
	Therefore		
	$\frac{d-b}{a} = \left(\frac{c-a}{a}\right)\left(-\frac{a}{d}\right)$	**A1**	
	$d^2 - bd = a^2 - ac$		
7	The result to be proved gives		
	$x_1 = 2 + 1 = 3$		
	which is correct so true for $n = 1$.	**B1**	
	Let the result be true for $n = k$, ie		
	$x_k = 2^k + k$	**M1**	
	Consider (for $n = k + 1$)		
	$x_{k+1} = 2(2^k + k) - k + 1$	**M1A1**	
	$= 2^{k+1} + (k+1)$	**A1**	
	Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$, the result is proved by induction.	**A1**	Award A1 for completely correct solution
8(a)	Taking logs,		
	$\ln f(x) = \sin x \ln x$	**M1**	
	Differentiating,		
	$\frac{f'(x)}{f(x)} = \cos x \ln x + \frac{\sin x}{x}$	**A1A1**	
	$f'(x) = (x)^{\sin x}(\cos x \ln x + \frac{\sin x}{x})$	**A1**	
(b)	Consider $f'(0.35) = -0.00451...$	**B1**	Accept – 0.00646…
	$f'(0.36) = 0.0156...$	**B1**	Accept 0.0223…
	The change of sign indicates a root between 0.35 and 0.36.	**B1**	

Ques	Solution	Mark	Notes
9(a)	$u + \mathrm{i}v = (x + \mathrm{i}[y+2])^2$ $= x^2 + 2\mathrm{i}x(y+2) - (y+2)^2$ Equating real and imaginary parts, $u = x^2 - (y+2)^2$; $v = 2x(y+2)$	**M1** **A1** **M1** **A1**	
(b)	Substituting $y = x - 1$, $u = x^2 - (x+1)^2 = -(2x+1)$ $v = 2x(x+1)$ Eliminating x, $v = -(u+1)\left(-\frac{(u+1)}{2}+1\right)$ $= \left(\frac{u^2-1}{2}\right)$ or equivalent	**M1** **A1** **M1** **A1**	FT from (a) provided equally difficult